A REVIEW OF

BY

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PREFAC

IN most high schools the course
finished by the end of the second y
most students have forgotten many
thorough review is necessary in orde
dates for the entrance examinations
the freshman year in college. Rec
schools are devoting at least two per
senior year to a review of algebra.

For such a review the regular text
an embarrassment of riches the tea

The definitions given i
reviewed as occasion arises
can be profitably employed
the part of the Outline tha
the example, or the formul

The whole scheme of th
of problems represent a da
apply to the Outlines or th
covered more rapidly. By
omissions indicated in the
algebra can be readily cov
thirty-two lessons, thus le
eighteen weeks, of two per

If a brief course is de
(pp. 31 to 35, 50 to 52), m
the book, and the Colleg
omitted without marring t
ness of the review.

CONTENTS

ᴊTLINE OF ELEMENT
INTERMEDIATE AI

Factors; coefficient; exponent;
term; algebraic sum; similar term
mogeneous expression; linear equat
equation; root of an expression;
ditional equation; prime quantity
mon factor (H. C. F.); lowest con
(L. C. M.); involution; evolutio
number; real number; rational; si
binomial surd; pure quadratic equ
quadratic equation; equation in
form; simultaneous linear equati
neous quadratic equations; discri

Special Rules for Multiplication and Division (*continued*)

5. Product of two bi... ing terms are sim...

$$(3x + 2 \ldots$$

6. Square of a polyno...

$$\left(m - \ldots\right.$$

7. Sum of two cubes.

8. Difference of two c...

$$\frac{x^3 - \ldots}{x - \ldots}$$

9. Sum or difference ...

$$\frac{x^7 + y^7}{x + y}, \frac{x^5 \ldots}{x \ldots}$$

1. Common monomial...

$$mx + my - \ldots$$

2. Trinomial that is a...

$$x^2 \pm 2x\ldots$$

Cases in Factoring (*continued*)

6. Sum or difference of
   - two cubes. See "
   - two like powers.
7. Common polynomial

$$t^2p + t^2q - 2\,mp -$$
$$= t^2(p + q) - 2\,m(p$$

8. Factor Theorem.

$$x^3 + 17\,x - 1$$

H. C. F. and L. C. M.

$a^2 + 2\,a - 3 = (a + 3)(a$

$a^2 + 7\,a + 12 = (a + 3)(a$

$a^4 + 27\,a = a(a + 3)(a^2 -$

H. C. F. $= a + 3$.

L. C. M. $= (a + 3)(a - 1)$

Fractions

Reduction to lowest term

Reduction of a mixed fraction.

Reduction of an improp number.

Addition and subtractio

Evolution

- Law of signs.
- Evolution of m
- Square root of a
- Square root of a
- Optional
  - Cub
  - Cub

Theory of Exponents

- Proofs: $a^m \cdot a^n$
- $\sqrt[n]{a^{mn}}$
- Meaning of
  - fr
  - ze
  - n
- T
- T

Radicals (*continued*)

- Multiplication and division of radicals
- Rationalization
  - Monon
  - Binom
  - Trinon
- Square root of a binomia
- Radical equations. *Alw*
  extraneous roots.

- Pure. $x^2 = a.$
- Affected. $ax^2 + bx +$
- Methods of solving
  - Co
  - Fo
  - Fa
- Equations in the quadra

Simultaneous Quadratics

Case I. One e…
The c…

Case II. Both
the

Case III. Any
x^2 –
x^2 ±

Case IV. Both
me
onc
oth

Ratio and Proportion

- Proportionals
  - mean,
  - third,
  - fourth.
- Theorems
  1. Product of
     of extrem
  2. If the pro
     equals th
     numbers,
  3. Alternation
  4. Inversion.
  5. Compositio
  6. Division.
  7. Compositio
  8. In a series
     of the an
     of the co
     cedent, et
- Special method of proving
  portion. Let $\frac{a}{b} = x$, $a =$

Development of formulas

A REVIEW OF

ORDER OF OPERATIONS, EVALU

Order of operations:

First of all, raising to a power a

Next, multiplication and divisio

Last of all, addition and subtra

Find the value of:

1. $5 \cdot 2^2 - \sqrt{25} \div 5 + 2^2 \cdot 8 \div 4 -$

2. $\dfrac{3 \times 6 \div 9}{2} - 2\sqrt{100} \div 5 + 4 \cdot 2$

12/14/22 3. $9 \cdot 2 \div 6 + 3 - 2 \cdot 4^2 \div \sqrt[3]{8} - 4$

Evaluate:

4. $\dfrac{a^4 - a^3 + b^3}{\sqrt{a^2b^2}} + \dfrac{c\sqrt{a} + a^3bc}{abc}$, if a

Give results by inspection:

1. $(g+\frac{1}{2}k)^2$. 9

2. $\left(s-\dfrac{2m}{3}\right)^2$.

10

3. $(2v+3w)(2v-3w)$.

4. $(x+3ts)(x-7ts)$. 11

5. $(2l+3g)(4l-11g)$.

12

6. $\left(a-\dfrac{2b}{3}+c-d\right)^2$.

13

7. $\dfrac{x^3+8m^3}{x+2m}$. 14

8. $\dfrac{y^3-27k^{3m}}{y-3k^m}$. 15

16. $(k^{32}+1)(k^{16}+1)(k^8+1)(k^4+1$

CASES IN

The number of terms in
clue to the possible cases und
ing the *test* for each and elim
one, the right case is readil
terms in the expression and
of the Cases in Factoring are
vitally important part of alge

CASE I. A common monon
ber of terms.

$5\,cx$

=

CASE II. A trinomial th
terms.

$x^2 \pm$

CASE III. The difference

$$c^4 + c^2d^2 + d^4 = c$$
$$= ($$
$$= ($$

CASE IV. A trinomial of the form

$$x^2 + x - 30 =$$

CASE V. A trinomial of the fo
terms.

$$20\,x^2 + 7\,x -$$

CASE VI. *A*. The sum or differe
terms.

$$x^3 + y^3 = (x +$$
$$x^3 - y^3 = (x -$$

B. The sum or differe
Two terms.

Review the *Cases* i
pages) and write out th

1. $8\,a^{13} + am^{12}$.

2. $x^7 + y^7$.

3. $4\,x^2 + 11\,x - 3$.

4. $m^2 + n^2 - (1 + 2\,mn$

5. $-\,x^2 + 2\,x - 1 + x^4$

6. $x^{16} - y^{16}$. (Five fa

7. $(x + 1)^2 - 5\,x - 29$.

8. $x^4 + x^2y^2 + y^4$.

9. $x^4 - 11\,x^2 + 1$.

10. $x^{2m} + 2 + \dfrac{1}{x^{2m}}$.

121

21. $gt - gk + gl^2$

22. $(m - n)(2\,a^2$

HIGHEST COMMON FACTOR A
MULTIPL

Define H. C. F. and L. C. M.

Find by factoring the H. C. F. and

1. $3x^2 - 3x$,
   $12x^2(x^2 - 1)$,
   $18x^3(x^3 - 1)$.

5.

2. $(x^2 - 1)(x^2 + 5x + 6)$,
   $(x^2 + 3x)(x^2 - x - 6)$.
   (*Harvard.*)

6.

3. $x^2 - y^2$,
   $x^2 + y^2$,
   $x^3 + y^3$,
   $x^6 + y^6$,
   $x^6 - y^6$.
   (*College Entrance Board.*)

7.

4. $x^3 + x^2 - 2$,

8.

Define: fraction, terms of a fra

Look up *the law of signs* as it a for this, fractions in algebra are they are in arithmetic.

1. Reduce to lowest terms:

(a) $\frac{32}{24}$; (b) $\frac{a^6-x^6}{a^4-x^4}$; (c) $\frac{(a+}{(a+}$

2. Reduce to a mixed express

3. Reduce to an improper frac

(a) $45\frac{1}{8}$; (b) $9\frac{11}{12}$ qt.; (c)

Add:

4. $\frac{5}{18}+\frac{7}{9}+\frac{11}{16}+\frac{5}{8}$.

5.

6. $\frac{1}{x(x-a)(x-b)}+\frac{1}{a(a-x)(}$

Multiply:

7. $\frac{72}{121}\times\frac{55}{56}\times\frac{77}{90}$.

COMPLEX FRACTIONS AND FRA

Define a complex fraction.

Simplify:

1. $\dfrac{\frac{3}{7}+\frac{4}{5}}{2-\frac{3}{7}\cdot\frac{4}{5}}$.

2. $\dfrac{2-\frac{3}{2}+\frac{2}{3}}{5-\frac{2}{3}+\frac{3}{2}}$.

4. $\dfrac{a}{b^2}-\dfrac{a}{b^2+\dfrac{cb}{a-\frac{c}{b}}}$. (*Harvard.*)

5. If $m=\dfrac{1}{a+1}$, $n=\dfrac{2}{a+2}$, $p=$

$\dfrac{m}{1-m}+\dfrac{n}{1-n}+\dfrac{p}{1-p}$?

6. Simplify the expression

$\left\{x+y-\dfrac{1}{x+y-\dfrac{xy}{\ldots}}\right\}\dfrac{x^3-\ldots}{x^2-\ldots}$

FRACTION

1. Solve for each letter in

2. Solve and check:

$$\frac{5x+2}{3}-\left(3-\frac{3x-1}{2}\right.$$

3. Solve and check:

$$\frac{1}{2}\left(x-\frac{a}{3}\right)-\frac{1}{3}\left(x-\frac{a}{4}\right)+$$

4. Solve (after looking up

$$\frac{3x-1}{30}+\frac{4x-7}{15}=\frac{x}{4}$$

5. Solve by the special *sh*

$$\frac{1}{x-2}-\frac{1}{x-3}=\frac{1}{x-4}$$

6. At what time between watch (*a*) opposite each oth gether?

Work out (*a*) and state the

SIMULTANEOUS EQUATIO

NOTE. Up to this point each topic presented h
xtent the preceding topics. For example, factori
les of multiplication and division; H. C. F. and L
ng; addition and subtraction of fractions and fracti
. C. F. and L. C. M., etc. From this point on, how
nce is not so marked, and miscellaneous example
lready covered will be given very frequently in o
ubject fresh in mind.

1. Solve by three methods — addition and
ution, and comparison: $\begin{cases} 5x + y = 11, \\ 3x + 2y = 1. \end{cases}$

Solve and check:

2. $\begin{cases} 12R_1 - 11R_2 = b + 12c, \\ R_1 + R_2 = 2b + c. \end{cases}$ **3.** $\begin{cases} \dfrac{r-s}{2} = \\ \dfrac{r+s-}{2} \end{cases}$

4. One half of A's marbles exceeds one h
ogether by 2; twice B's marbles falls sh
ogether by 16; if C had four more marbl

1. Solve $\begin{cases} \frac{3}{4x} - \frac{5}{3y} = 11\frac{1}{2}, \\ \frac{5}{8x} - \frac{3}{2y} = 10\frac{1}{4}. \end{cases}$ I ing the clea

2. Solve $\begin{cases} \frac{1}{x} - \frac{1}{y} - \frac{1}{z} \\ \frac{1}{y} - \frac{1}{z} - \frac{1}{x} \\ \frac{1}{z} - \frac{1}{x} - \frac{1}{y} \end{cases}$

3. Solve graphically and algebr

4. Solve graphically and algebr

Review:

5. The squares of the numbers

6. The cubes of the numbers fr

7. The fourth powers of the nu

SQUARE R

Find the square root of:

1. $$1 + 16\,m^6 - 40\,m^4 + 10\,m$$

2. $$\frac{a^2}{x^2} + \frac{6\,a}{x} + 11 -$$

3. Find the square root to thre

4. Find the square root of 337,

5. Find the square root of 182

6. Find to four decimal places

7. Add $\quad \dfrac{2}{(x-1)^3} + \dfrac{1}{(1-x)^2}$

8. Find the value of:

$$\frac{\sqrt[3]{64} \cdot 12}{24} \div 2 \times 3 - \frac{2 \cdot 7^2}{14} \div 7$$

9. Simplify $[(x+y)^5 + (x-y)^5]$

10. Solve by the short method:

Review the proofs, for positi

I. $a^m \times a^n = a^{m+n}$.

II. $$\frac{a^m}{a^n} = a^{m-n}.$$

III. $(a^m)^n = a^{mn}$.

To find the meaning of a fract

Assume that Law I holds fo

If so, $a^{\frac{2}{3}} \cdot a^{\frac{2}{3}} \cdot a$

Hence, $a^{\frac{2}{3}}$ is *one of the three e*

of a^2.

$\therefore a$

In the same way, $a^{\frac{4}{5}} \cdot a^{\frac{4}{5}} \cdot a^{\frac{4}{5}}$

Hence, $a^{\frac{4}{5}}$ is *one of the five e*

of a^4.

$\therefore a$

In the same way, in general, a

Hence, *the numerator of a*

THEORY OF EXPONENTS

Rules:

To multiply quantities having the same

To divide quantities having the same b

To raise a quantity to a power, multipl

To extract a root, divide the exponent o
of the root.

1. Find the value of $3^2 - 5 \times 4^0 +$

2. Find the value of $8^{-\frac{2}{3}} + 9^{\frac{3}{2}} - 2^{-}$

Give the value of each of the followin

3. $\frac{3^0}{5}, \frac{3}{5^0}, \frac{3^0}{5^0}, 3^0 \times 5, 3 \times 5^0, 3^0 \times 5^0$

4. Express 7^0 as some power of 7 di

Simplify:

5. $16^{\frac{1}{3}} \cdot 2^{\frac{1}{2}} \cdot 32^{\frac{5}{6}}$. (Change t

Solve for x:

1. $x^{\frac{2}{3}} = 4.$

Factor:

3. $x^{\frac{2}{3}} - 9.$

4. $x^{\frac{3}{5}} + 27.$

7. Find the H. C. F. and L.

$$a^2 + a^{\frac{3}{2}}b^{\frac{1}{2}} + a^{\frac{1}{2}}b^{\frac{3}{2}} - b^2,$$

8. Simplify the product of

$$(ayx^{-1})^{\frac{1}{2}},\ (bxy^{-2})^{\frac{1}{3}}, \text{ and}$$

9. Find the square root of:

$$25\,a^{\frac{4}{3}}b^{-3} - 10\,a^{\frac{2}{3}}b^{-\frac{3}{2}} - 4$$

10. Simplify $\sqrt[5]{\frac{2^{n+2}}{4^{-n}} \div \frac{8^n}{2^3}}.$

RADICALS

1. Review all definitions in Radicals, transforming and simplifying radicals. *its simplest form?*

2. Simplify (to simplest form): $\sqrt{\frac{2}{3}}$

$\frac{2a}{b}\sqrt{\frac{8b^2}{27a}}$; $\sqrt[2n]{\frac{5}{x^n}}$; $(a+b)^2\sqrt[3]{\frac{-a^4}{(a+b)^5}}$; $\sqrt{2}$

3. Reduce to entire surds: $2\sqrt{3}$;

$-3\sqrt[3]{2}$; $3a\sqrt[3]{\frac{a+2}{6a^2}}$; $(a+2y)\sqrt{\frac{a-2y}{a+2y}}$

4. Reduce to radicals of lower order

$\sqrt[4]{a^2}$; $\sqrt[6]{a^3}$; $\sqrt[6]{27a^3}$; $\sqrt[12]{81a^4x^8}$

5. Reduce to radicals of the same deg

$\sqrt{7}$ and $\sqrt[3]{11}$; $\sqrt[3]{5}$ and $\sqrt[4]{3}$; $\sqrt[6]{7}$ and

$\sqrt[x]{c^y}$, $\sqrt[y]{c^z}$, and $\sqrt[z]{c^x}$.

6. Which is greater, $\sqrt{3}$ or $\sqrt[3]{4}$?

7. Which is greatest,

RADICALS (

The most important principle i

$(ab)^{\frac{1}{n}} = a^{\frac{1}{n}}b^{\frac{1}{n}}$. Hence $\sqrt[n]{ab} = \sqrt[n]{a}$

From this also $\dfrac{\sqrt[n]{ab}}{\sqrt[n]{a}} = \sqrt[n]{b}$

Multiply:

1. $2\sqrt[3]{4}$ by $3\sqrt[3]{6}$.
2. $\sqrt{2}$ by $\sqrt[3]{3}$.
5. $\sqrt{2}+\sqrt{3}-\sqrt{5}$ by $\sqrt{2}-\sqrt{}$
6. $-\dfrac{p}{2}+\dfrac{\sqrt{p^2-4q}}{2}$ by $-\dfrac{p}{2}-$

Divide:

7. $\sqrt{27}$ by $\sqrt{3}$.
8. $4\sqrt{18}$ by $5\sqrt{32}$.
11. $6\sqrt{105}+18\sqrt{40}-45\sqrt{12}$
12. $10\sqrt[3]{18}-4\sqrt[3]{60}+5\sqrt[3]{100}$ b

MISCELLANEOUS EXAMPLES,
TO QUADRATICS

Results by inspection, examples 1–10.

Divide:

1. $\dfrac{x^{\frac{5}{17}} + y^{\frac{5}{17}}}{x^{\frac{1}{17}} + y^{\frac{1}{17}}}$.

2. $\dfrac{x - y}{x^{\frac{1}{3}} - y^{\frac{1}{3}}}$.

3. $\dfrac{m^2 + n^2}{m^{\frac{2}{3}} + n^{\frac{2}{3}}}$.

4. $\dfrac{x - y^2}{\sqrt[3]{x} - \sqrt[3]{y^2}}$.

Multipl

5. $\left(a^{-\frac{3}{4}} \right.$

6. $(K^{-\frac{2}{7}}$

7. $(r^{2s} +$

8. $\left(a^{-2} - \right.$

9. $(3K^{x}$

10. $(2y^{\frac{2}{7}} -$

Factor:

11. $x^{\frac{2}{3}} - 64$.

12. $y^{\frac{3}{5}} + 27$.

13. $b^{\frac{3}{2}} - 8$

14. $3p -$

Factor, using radicals instead of expone

3. Find the square root of 8114.4061. What, then, is the square root of .0081144061? of 811440.61? From any of the above can you determine the square root of .081144061?

4. The H.C.F. of two expressions is $a(a-b)$, and their L.C.M. is $a^2b(a+b)(a-b)$. If one expression is $ab(a^2-b^2)$, what is the other?

5. Solve (short method):

$$\frac{5}{7-x} - \frac{2\frac{1}{4}x-3}{4} - \frac{x+11}{8} + \frac{11x+5}{16} = 0.$$

6. Solve
$$\begin{cases} \dfrac{2}{m} - \dfrac{3}{n} + \dfrac{10}{p} = -3, \\ \dfrac{4}{m} + \dfrac{5}{p} + \dfrac{6}{n} = 15, \\ \dfrac{1}{m} - \dfrac{1}{n} + \dfrac{5}{p} = -\dfrac{1}{2}. \end{cases}$$

7. Simplify $21\sqrt{\frac{3}{8}} - 5\sqrt{\frac{1}{6}} + 6\sqrt{4\frac{1}{6}} - 10\sqrt{3\frac{1}{6}} + \frac{40}{3}\sqrt{1\frac{1}{2}}$.

8. Does $\sqrt{16 \times 25} = 4 \times 5$? Does $\sqrt{16+25} = 4+5$?

9. Write the fraction $\dfrac{5}{4+2\sqrt{3}}$ with rational denominator, and find its value correct to two decimal places.

10. Simplify $$\frac{\left\{\sqrt{\dfrac{p+\sqrt{p^2-q}}{2}} + \sqrt{\dfrac{p-\sqrt{p^2-q}}{2}}\right\}^2}{p+\sqrt{q}}.$$

(*Princeton.*)

Simplify $\dfrac{2^{n+1} - 2(2^n)}{2(2^{n+3})}$. *(Univ. of Penn.)*

Find the value of $\dfrac{1 + 8^{-\frac{2}{3}}}{(8x)^{\frac{1}{3}} + 10^{x-2}}$, when $x = 2$. *(Cornell.)*

Find the value of x if $\begin{cases} x^{\frac{3}{2}} = y^4, \\ y^{\frac{2}{3}} = 9. \end{cases}$ *(M. I. T.)*

A fisherman told a yarn about a fish he had caught. If
ish were half as long as he said it was, it would be 10
es more than twice as long as it is. If it were 4 inches
r than it is, and he had further exaggerated its length
lding 4 inches, it would be $\frac{1}{8}$ as long as he now said it
How long is the fish, and how long did he first say it
(M. I. T.)

The force P necessary to lift a weight W by means of a
in machine is given by the formula

$$P = a + bW,$$

e a and b are constants depending on the amount of fric-
n the machine. If a force of 7 pounds will raise a weight
pounds, and a force of 13 pounds will raise a weight of 50
ls, what force is necessary to raise a weight of 40 pounds?
t determine the constants a and b.) *(Harvard.)*

Reduce to the simplest form: $\sqrt[n]{\dfrac{4}{2^{n+2}}}$, $\dfrac{ax(a^{-1}x - ax^{-1})}{x^{\frac{1}{2}} - a^{\frac{1}{2}}}$.

Determine the H.C.F. and L.C.M. of $(xy - y^2)^3$ and
^2y. *(College Entrance Board.)*

$$\frac{\left(a^{\frac12}+\frac{1}{x^{-\frac12}}\right)^2-\left(\frac{1}{a^{-\frac12}}-x^{\frac12}\right)^2}{x+\sqrt{a^2+x^2}}.$$ (*M. I. T.*)

3. Find $\sqrt{7-\sqrt{48}}$.

4. Expand $(\sqrt{a^3}-\sqrt{b^6})^6$.

5. Expand and simplify $(1-2\sqrt{3}+3\sqrt{2})^2$.

6. Solve the simultaneous equations $\begin{cases} x^{-\frac12}+2y^{-\frac12}=\frac76, \\ 2x^{-\frac12}-y^{-\frac12}=\frac23. \end{cases}$ $x=4$ $y=9$

(*Yale.*)

7. Find to three places of decimals the value of

$$\sqrt{\frac{(a+b)^{\frac13}}{(11a+b^2)^{\frac16}}\cdot\frac{(a^3-b^3)^{\frac12}}{(a-b)^{\frac13}}},$$

when $a=5$ and $b=3$. (*Columbia.*)

8. Show that $\dfrac{10-4\sqrt{5}}{5+3\sqrt{5}}$ is the negative of the reciprocal of $\dfrac{10+4\sqrt{5}}{5-3\sqrt{5}}$. (*Columbia.*)

9. Solve and check $\dfrac{5}{\sqrt{3x+2}}=\sqrt{3x+2}+\sqrt{3x-1}$. $x=1$

10. Assuming that when an apple falls from a tree the distance (S meters) through which it falls in any time (t seconds) is given by the formula $S=\frac12 gt^2$ (where $g=9.8$), find to two decimal places the time taken by an apple in falling 15 meters. 1.74

(*College Entrance Board.*)

Arithmetic

$$p = br \qquad a = p + prt$$

$$i = prt$$

Geometry

$$K = \tfrac{1}{2}\, bh \qquad V = \pi R^2 H$$

$$K = bh \qquad V = \tfrac{1}{3}\, \pi R^2 H$$

$$K = \frac{a^2}{4}\sqrt{3} \qquad V = \tfrac{4}{3}\, \pi R^3$$

$$K = \tfrac{1}{2}\,(b + b')\, h \qquad S = \frac{\pi R^2 E}{180}$$

$$K = \pi R^2 \qquad \frac{C}{C'} = \frac{R}{R'}$$

$$C = 2\, \pi R$$

$$K = \pi R L \qquad \frac{K}{K'} = \frac{R^2}{R'^2}$$

$$S = 4\, \pi R^2$$

Physics

$$v = gt$$

$$s = \tfrac{1}{2}\, gt^2 \qquad t = \pi\sqrt{\frac{l}{g}}$$

$$s = \frac{v^2}{2\,g} \qquad F = \frac{mV^2}{r}$$

$$C = \frac{E}{R} \qquad mh = \frac{mv^2}{2\,g}$$

$$E = \frac{wv^2}{2\,g} \qquad R = \frac{gs}{g + s}$$

$$e = \frac{4\,Pl^3}{bh^3 m} \qquad E = \frac{4\,\pi^2 l^2 w}{g}$$

$$E = \frac{mv^2}{2} \qquad C = \tfrac{5}{9}\,(F - 32)$$

Review the first (or usual) method of completing the square. Solve by it the following:

3. $x^2 + 10x = 24$.

4. $2x^2 - 5x = 7$.

5. $\dfrac{x-1}{2} + \dfrac{2}{x-1} = 2\frac{1}{2}$.

6. $ax^2 + bx + c = 0$.

Review the solution by factoring. Solve by it the following:

7. $x^2 + 8x + 7 = 0$.

8. $24x^2 = 2x + 15$.

9. $3 = 10x - 3x^2$.

10. $-7 = 6x - x^2$.

Solve, by factoring, these equations, which are not quadratics:

11. $x^4 = 16$.

12. $x^3 = 8$.

13. $x^3 = x$.

Review the solution by formula. Solve by it the following:

14. $5x^2 - 6x = 8$.

15. $\frac{1}{2}(x+1) - \frac{x}{3}(2x-1) = -12$.

16. $x^2 + 4ax = 12a^2$.

17. $3x^2 = 2rx + 2r^2$.

Solve graphically:

18. $x^2 - 2x - 8 = 0$.

19. $x^2 + x - 2 = 0$.

Reference: The chapter on Quadratic Equations in any algebra (first part of the chapter).

$x^4 - 5x^2 = -4.$ 3. $2\sqrt[3]{x^{-2}} - 3\sqrt[3]{x^{-1}} = 2.$

$\frac{x+3}{x-3} + 6 = 5\sqrt{\frac{x+3}{x-3}}.$ $\left(\text{Let } y = \sqrt{\frac{x+3}{x-3}} \text{ and substitute.}\right)$

$3x^2 - 4x + 2\sqrt{3x^2 - 4x - 6} = 21.$

$x^2 + 5x - 5 = \frac{6}{x^2 + 5x}.$

ve and check:

$\sqrt{x+7} + \sqrt{3x-2} = \frac{4x+9}{\sqrt{3x-2}}.$

$\sqrt{x^2-5} + \frac{6}{\sqrt{x^2-5}} = 5.$

$\frac{10w}{\sqrt{10w-9}} - \sqrt{10w+2} = \frac{2}{\sqrt{10w-9}}.$

e results by inspection:

$(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}).$

$(\sqrt{10} + \sqrt{19})(\sqrt{10} - \sqrt{19}).$

How many gallons each of cream containing 33 % fat and milk containing 6 % butter fat must be mixed duce 10 gallons of cream containing 25 % butter fat?

I have \$6 in dimes, quarters, and half-dollars, there being ns in all. The number of dimes and quarters together is mes the number of half-dollars. How many coins of ind are there? *(College Entrance Board.)*

nce: The last part of the chapter on Quadratic Equations in any algebra.

Or, $$x^2 + \frac{b}{a}x + \frac{c}{a} = 0. \qquad (2)$$

To derive the formula, we have by transposing

$$x^2 + \frac{b}{a}x = -\frac{c}{a}.$$

Completing the square,

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2 - 4ac}{4a^2}.$$

Extracting square root, $x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}.$

Transposing, $$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}.$$

Hence, $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

These two values of x we call *roots*.

For convenience represent them by r_1 and r_2.

Hence, $$r_1 = -\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}.$$

$$r_2 = -\frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}.$$

Adding, $$r_1 + r_2 = -\frac{2b}{2a} = -\frac{b}{a}. \qquad (3)$$

$$= \frac{b^2 - b^2 + 4ac}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a}. \qquad (4)$$

ence we have shown that $\begin{cases} r_1 + r_2 = -\dfrac{b}{a}, \\ \text{and } r_1 r_2 = \dfrac{c}{a}. \end{cases}$

, referring to equation (2) above, we have the following rule:

hen the coefficient of x^2 is unity, the sum of the roots is the cient of x with the sign changed; the product of the roots is ndependent term.

XAMPLES:

$x^2 - 9x + 21 = 0.$ $\begin{cases} \text{Sum of the roots} = 9. \\ \text{Product of the roots} = 21. \end{cases}$

$3x^2 - 7x - 18 = 0.$ $\begin{cases} \text{Sum of the roots} = \frac{7}{3}. \\ \text{Product of the roots} = -6. \end{cases}$

$-21x = 17 - 4x^2.$ $\begin{cases} \text{Sum of the roots} = \frac{21}{4}. \\ \text{Product of the roots} = -\frac{17}{4}. \end{cases}$

. To find the nature or character of the roots.

s before, $$r_1 = -\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a},$$

$$r_2 = -\frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}.$$

he $\sqrt{b^2 - 4ac}$ determines the *nature* or *character* of the s; hence it is called the *discriminant*.

1. $x^2 - 4x + 2 = 0$.

$\sqrt{b^2 - 4ac} = \sqrt{16 - 8} = \sqrt{8}$. $\therefore$ The roots are real, unequal, and irrational.

2. $x^2 - 4x + 6 = 0$.

$\sqrt{b^2 - 4ac} = \sqrt{16 - 24} = \sqrt{-8}$. $\therefore$ The roots are imaginary and unequal.

3. $x^2 - 4x + 4 = 0$.

$\sqrt{b^2 - 4ac} = \sqrt{16 - 16} = \sqrt{0}$. $\therefore$ The roots are real, equal, and rational.

III. **To form the quadratic equation when the roots are given.**

Suppose the roots are 3, -7.

Then, $\begin{cases} x = 3, \\ x = -7. \end{cases}$ Or, $\begin{aligned} x - 3 &= 0, \\ x + 7 &= 0. \end{aligned}$

Multiplying to get a quadratic, $(x-3)(x+7) = 0$.

Or, $x^2 + 4x - 21 = 0$.

Or, use the sum and product idea developed on the preceding page. The coefficient of x^2 must be unity.

Add the roots and change the sign to get the coefficient of x.

Multiply the roots to get the independent term.

$\therefore$ The equation is $x^2 + 4x - 21 = 0$.

In the same way, if the roots are $\frac{2+\sqrt{3}}{7}$, $\frac{2-\sqrt{3}}{7}$, the equation is

$$x^2 - \tfrac{4}{7}x + \tfrac{1}{49} = 0.$$

$x - 3$

2. $9x^2 - 6x + 1 = 0.$

3. $x^2 + 2x + 9734 = 0.$

4. $16 + \frac{5}{x} = \frac{17}{x^2}.$

6. $(x + 7)(x - 6) = 70.$

7. $x^2 - x\sqrt{2} = 3.$

8. $pr^2 + qr + s = 0.$

orm the equations whose roots are:

9. $5, -3.$

0. $\frac{3}{8}, \frac{5}{8}.$

1. $c + d, c - d.$

2. $-3, -5.$

13. $\frac{2 \pm \sqrt{-3}}{5}.$

14. $\frac{8}{3} + \frac{2}{3}\sqrt{37}, \frac{8}{3} - \frac{2}{3}\sqrt{37}.$

15. $\frac{-2 \pm \sqrt{-2}}{2}.$

3. Solve $x^2 - 3x + 4 = 0.$ Check by substituting the es of x; then check by finding the sum and the product of roots. Compare the amount of labor required in each case.

7. Solve $(x - 3)(x + 2)(x^2 + 3x - 4) = 0.$

3. Is $e^{4x} + 2e^{3x} + e^{2x} + 2e^x + 2 + e^{-2x}$ a perfect square?

9. Find the square root (short method):

$$(x^2 - 1)(x^2 - 3x + 2)(x^2 - x - 2).$$

0. Solve $\frac{1.2x - 1.5}{1.5} + \frac{.4x + 1}{.2x - .2} = \frac{.4x + 1}{.5}.$

. The glass of a mirror is 18 inches by 12 inches, and it a frame of uniform width whose area is equal to that of glass. Find the width of the frame.

Method: Solve for x in terms of y, or *vice versa*, in the linear and substitute in the quadratic.

Simultaneous Quadratics

Case II. Both equations homogeneous and of the second degree.

$$\begin{cases} x^2 - xy + y^2 = 39, \\ 2x^2 - 3xy + 2y^2 = 43. \end{cases}$$

Method: Let $y = vx$, and substitute in both equations.

Alternate Method: Solve for x in terms of y in one equation and substitute in the other.

Case III. Any two of the quantities $\left\{\begin{matrix} x + y \\ x^2 + y^2 \\ xy \\ x - y \\ x^3 + y^3 \\ x^3 - y^3 \\ x^2 + xy + y^2 \\ x^2 - xy + y^2 \end{matrix}\right\}$ given.

$$\begin{cases} x + y = 5, \\ x^2 - xy + y^2 = 7. \end{cases}$$

Method: Solve for $x + y$ and $x - y$; then add to get x, subtract to get y.

$\quad x + y = 2.$

METHOD: Let $x = u + v$ and $y = u - v$, and substitute in both equations.

Simultaneous Quadratics (continued)

Special Devices

I. Consider some compound quantity like xy, $\sqrt{x-y}$, $\sqrt{xy}$, $\frac{x}{y}$, etc., as the unknown, at first. Solve for the compound unknown, and combine the resulting equation with the simpler original equation.

$$\begin{cases} x^2y^2 + xy = 6, \\ x + 2y = -5. \end{cases}$$

II. Divide the equations member by member. Then solve by Case I, II, or III.

$$\begin{cases} x^3 - y^3 = 152, \\ x - y = 2. \end{cases}$$

III. Eliminate the quadratic terms. Then solve by Case I, II, or III.

$$\begin{cases} xy + x = 15, \\ xy + y = 16. \end{cases}$$

2. $\left\{\begin{array}{l} \ldots \\ xy + y^2 = 14. \end{array}\right.$

9. $\left\{\begin{array}{l} \ldots \\ 3x^2 + 2xy - y^2 = 3. \end{array}\right.$

3. $\begin{cases} x^2 + y^2 = 25, \\ x + y = 1. \end{cases}$

10. $\begin{cases} x^5 + y^5 = 242, \\ x + y = 2. \end{cases}$

4. $\begin{cases} x^4 + y^4 = 2, \\ x - y = 2. \end{cases}$

11. $\begin{cases} x - y + \sqrt{x - y} = 6, \\ xy = 5. \end{cases}$

5. $\begin{cases} x^3 + y^3 = 28, \\ x + y = 4. \end{cases}$

12. $\begin{cases} 4x^2 - x + y = 67, \\ 3x^2 - 3y = 27. \end{cases}$

6. $\begin{cases} x^2y^2 + xy - 12 = 0, \\ x + y = 4. \end{cases}$

13. $\begin{cases} x - y - \sqrt{x - y} = 2, \\ x^3 - y^3 = 2044. \end{cases}$ (*Yale.*)

7. $\begin{cases} 2xy - x + 2y = 16, \\ 3xy + 2x - 4y = 10. \end{cases}$

14. $\begin{cases} x^2 + xy + x = 14, \\ y^2 + xy + y = 28. \end{cases}$

(*Princeton.*)

15. $\begin{cases} x^2 + y^2 = 13, \\ y^2 = 4(x - 2). \end{cases}$ Plot the graph of each equation.

(*Cornell.*)

16. $\begin{cases} x^2 + y^2 = xy + 37, \\ x + y = xy - 17. \end{cases}$ (*Columbia.*)

In grouping the answers, be sure to associate each value of x with the corresponding value of y.

17. The course of a yacht is 30 miles in length and is in the shape of a right triangle one arm of which is 2 miles longer than the other. What is the distance along each side?

Reference: The chapter on Simultaneous Quadratics in any algebra.

Find a third proportional to 4 and 7; 5 and 10; a^2-9 $a-3$.

Find a fourth proportional to 2, 5, and 4; 35, 20, and 14.

Write out the proofs for the following, stating the em in full in each case:

The product of the extremes equals etc.

If the product of two numbers equals the product of two numbers, either pair etc.

Alternation. (e) Composition.

Inversion. (f) Division.

Composition and division.

In a series of equal ratios, the sum of the antecedents the sum of the consequents etc.

Like powers or like roots of the terms of a proportion etc.

If $x:m::13:7$, write all the possible proportions that e derived from it. [See (5) above.]

Given $rs = 161\,m$; write the eight proportions that may rived from it, and quote your authority.

(a) What theorem allows you to change any proportion an equation?

What theorem allows you to change any equation into a rtion?

If $xy=ry$, what is the ratio of x to g? of y to r? of y to g?

Find two numbers such that their sum, difference, and um of their squares are in the ratio $5:3:51$. (*Yale.*)

ence: The chapter on Ratio and Proportion in any algebra.

Let $\frac{a}{b} = r$. $\therefore a = br$.

Also $\frac{c}{d} = r$. $\therefore c = dr$.

Substitute the value of a in the first ratio, and c in the second:

$$\text{Then} \quad \frac{3a^3 + 5ab^2}{3a^3 - 5ab^2} = \frac{3b^3r^3 + 5b^3r}{3b^3r^3 - 5b^3r} = \frac{b^3r(3r^2 + 5)}{b^3r(3r^2 - 5)} = \frac{3r^2 + 5}{3r^2 - 5}.$$

$$\text{Also} \quad \frac{3c^3 + 5cd^2}{3c^3 - 5cd^2} = \frac{3d^3r^3 + 5d^3r}{3d^3r^3 - 5d^3r} = \frac{d^3r(3r^2 + 5)}{d^3r(3r^2 - 5)} = \frac{3r^2 + 5}{3r^2 - 5}.$$

$$\therefore \quad \frac{3a^3 + 5ab^2}{3a^3 - 5ab^2} = \frac{3c^3 + 5cd^2}{3c^3 - 5cd^2}. \qquad \text{Axiom 1.}$$

Or, $3a^3 + 5ab^2 : 3a^3 - 5ab^2 = 3c^3 + 5cd^2 : 3c^3 - 5cd^2$.

If $a : b = c : d$, prove:

1. $a^2 + b^2 : a^2 = c^2 + d^2 : c^2$.
2. $a^2 + 3b^2 : a^2 - 3b^2 = c^2 + 3d^2 : c^2 - 3d^2$.
3. $a^2 + 2b^2 : 2b^2 = ac + 2bd : 2bd$.
4. $2a + 3c : 2a - 3c = 8b + 12d : 8b - 12d$.
5. $a^2 - ab + b^2 : \frac{a^3 - b^3}{a} = c^2 - cd + d^2 : \frac{c^3 - d^3}{c}$.

6. The second of three numbers is a mean proportional between the other two. The third number exceeds the sum of the other two by 20; and the sum of the first and third exceeds three times the second by 4. Find the numbers.

7. Three numbers are proportional to 5, 7, and 9; and their sum is 14. Find the numbers. (*College Entrance Board.*)

8. A triangular field has the sides 15, 18, and 27 rods, respectively. Find the dimensions of a similar field having 4 times the area.

$$\begin{cases} S = \dfrac{n}{2}(a + l), \\ \\ S = \dfrac{n}{2}[2a + (n-1)d]. \end{cases}$$

. Find the sum of the first 50 odd numbers.

. In the series 2, 5, 8, ⋯, which term is 92?

. How many terms must be taken from the series 3, 5, 7, make a total of 255?

. Insert 5 arithmetical means between 11 and 32.

Insert 9 arithmetical means between $7\frac{1}{2}$ and 30.

. Find x, if $3 + 2x$, $5 + 6x$, $9 + 5x$ are in A. P.

. The 7th term of an arithmetical progression is 17, and 3th term is 59. Find the 4th term.

. How can you turn an A. P. into an equation?

. Given $a = -\frac{5}{3}$, $n = 20$, $S = -\frac{5}{3}$, find d and l.

. Find the sum of the first n odd numbers.

An arithmetical progression consists of 21 terms. The of the three terms in the middle is 129; the sum of the hree terms is 237. Find the series. (Look up the short od for such problems.) (*Mass. Inst. of Technology.*)

. B travels 3 miles the first day, 7 miles the second day, iles the third day, etc. In how many days will B over- A who started from the same point 8 days in advance and travels uniformly 15 miles a day?

ence: The chapter on Arithmetical Progression in any algebra.

II. $S = \dfrac{ar^n - a}{r - 1}.$ IV. $S_\infty = \dfrac{a}{1 - r}.$

2. How many terms must be taken from the series 9, 18, 36, … to make a total of 567?

3. In the G. P. 2, 6, 18, …, which term is 486?

4. Find x, if $2x - 4$, $5x - 7$, $10x + 4$ are in geometrical progression.

5. How can you turn a G. P. into an equation?

6. Insert 4 geometrical means between 4 and 972.

7. Insert 6 geometrical means between $\frac{5}{16}$ and 5120.

8. Given $a = -2$, $n = 5$, $l = -32$; find r and S.

9. If the first term of a geometrical progression is 12 and the sum to infinity is 36, find the 4th term.

10. If the series $3\frac{1}{8}$, $2\frac{1}{2}$, … be an A. P., find the 97th term. If a G. P., find the sum to infinity.

11. The third term of a geometrical progression is 36; the 6th term is 972. Find the first and second terms.

12. Insert between 6 and 16 two numbers, such that the first three of the four shall be in arithmetical progression, and the last three in geometrical progression.

13. A rubber ball falls from a height of 40 inches and on each rebound rises 40% of the previous height. Find by formula how far it falls on its eighth descent. (Yale.)

Reference: The chapter on Geometrical Progression in any algebra.

$(x + x^{-1})^n$. 6. $(x^2 - x + 2)^3$.

$\left(\frac{a}{x} - \frac{x}{a}\right)^6$. 7. $\left(\frac{2\sqrt[3]{b^2}}{y} + \frac{3\sqrt{y}}{b^3}\right)^4$.

$$(a + b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{1 \cdot 2}a^{n-2}b^2$$

$$+ \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}a^{n-3}b^3 + \frac{n(n-1)(n-2)(n-3)}{1 \cdot 2 \cdot 3 \cdot 4}a^{n-4}b^4 + \cdots.$$

ow by observation that the formula for the

$$(r + 1)\text{th term} = \frac{n(n-1)(n-2)\cdots(n-r+1)}{1 \cdot 2 \cdot 3 \cdot 4 \cdots r}a^{n-r}b^r.$$

. Indicate what the 97th term of $(a + b)^n$ would be.

. Using the expansion of $(a + b)^n$ in (8), derive a formula he rth term by observing how each term is made up, then ralizing.

ing either the formula in (8) or (10), whichever you are liar with, find:

. The 4th term of $\left(a + \frac{1}{a}\right)^{30}$.

. The 8th term of $(1 + x\sqrt{y})^{13}$.

. The middle term of $(2a^{\frac{1}{2}} - y\sqrt[3]{a})^{10}$.

. The term not containing x in $\left(x^3 - \frac{2}{x}\right)^{12}$.

. The term containing x^{18} in $\left(x^2 - \frac{a}{x}\right)^{15}$.

rence: The chapter on The Binomial Theorem in any algebra.

(*Sheffield Scientific School.*)

3. Solve $2\sqrt{2x+2}+\sqrt{2x+1}=\dfrac{12x+4}{\sqrt{8x+8}}$. (*Yale.*)

4. Solve the equation $V=\dfrac{H}{3}(B+x+\sqrt{Bx})$ for x, taking $H=6$, $B=8$, and $V=28$; and verify your result. (*Harvard.*)

5. Solve $\begin{cases} x:y=2:3, \\ x^3+y^2=5(x+y)+2. \end{cases}$

6. Solve $2x^2-4x+3\sqrt{x^2-2x+6}=15$. (*Coll. Ent. Board.*)

7. Find all values of x and y which satisfy the equations:

$$\begin{cases} \sqrt{x}+\sqrt{y}=4, \\ \dfrac{1}{\sqrt{x+1}-\sqrt{x}}-\dfrac{1}{\sqrt{x+1}+\sqrt{x}}=y. \end{cases}$$ (*Mass. Inst. of Technology.*)

8. If α and β represent the roots of $px^2+qx+r=0$, find $\alpha+\beta$, $\alpha-\beta$, and $\alpha\beta$ in terms of p, q, and r. (*Princeton.*)

9. Form the equation whose roots are $2+\sqrt{-3}$ and $2-\sqrt{-3}$.

10. Determine, without solving, the character of the roots of

$$9x^2-24x+16=0.$$ (*College Entrance Board.*)

11. If $a:b=c:d$, prove that $a+b:c+d=\sqrt{a^2+b^2}:\sqrt{c^2+d^2}$. (*College Entrance Board.*)

12. Given $a:b=c:d$. Prove that $a^2+b^2:\dfrac{a^3}{a+b}=c^2+d^2:\dfrac{c^3}{c+d}$. (*Sheffield.*)

13. The 9th term of an arithmetical progression is $\frac{1}{6}$; the 16th term is $\frac{5}{6}$. Find the first term. (*Regents.*)

the sum of the first two is 1, and the sum of the last two 19.

What number added to 2, 20, 9, 34, will make the ts proportional?

Find the middle term of $\left(3a^5 + \frac{b^{\frac{3}{4}}}{2}\right)^8$.

Solve $\dfrac{x+1}{3x+2} = \dfrac{2x-3}{3x-2} - 1 - \dfrac{36}{4-9x^2}$. (*Princeton.*)

A strip of carpet one half inch thick and $29\frac{6}{7}$ feet long led on a roller four inches in diameter. Find how many there will be, remembering that each turn increases the eter by one inch, and that the circumference of a circle s (approximately) $\frac{22}{7}$ times the diameter. (*Harvard.*)

The sum of the first three terms of a geometrical progres- is 21, and the sum of their squares is 189. What is the term? (*Yale.*)

Find the geometrical progression whose sum to infinity nd whose second term is $\frac{3}{4}$.

Solve $4x + 4\sqrt{3x^2 - 7x + 3} = 3x^2 - 3x + 6$.

Solve $\begin{cases} 2x^2 + 3xy - 5y^2 = 4, \\ \qquad 2xy + 3y^2 = -3. \end{cases}$

Two hundred stones are placed on the ground 3 feet , the first being 3 feet from a basket. If the basket and e stones are in a straight line, how far does a person l who starts from the basket and brings the stones to by one?

1. $\begin{cases} x^2 + y^2 = 25, \\ x + y = 1. \end{cases}$

2. $x^2 - 3x - 18 = 0.$ 3. $x^2 + 3x - 10 = 0.$

Determine the value of m for which the roots of the equation will be equal: (HINT: See page 40. To have the roots equal, $b^2 - 4ac$ must equal 0.)

4. $$2x^2 - mx + 12\tfrac{1}{2} = 0.$$

5. $$(m-1)x^2 + mx + 2m - 3 = 0.$$

6. If $2a + 3b$ is a root of $x^2 - 6bx - 4a^2 + 9b^2 = 0$, find the other root without solving the equation.

(*Univ. of Penn.*)

7. How many times does a common clock strike in 12 hours?

8. Find the sum to infinity of $\frac{2}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}, \ldots$.

9. Solve $$\left(\frac{x}{2}+\frac{6}{x}\right)^2 - 6\left(\frac{x}{2}+\frac{6}{x}\right) + 8 = 0.$$

10. Find the value of the recurring decimal 2.214214 ....

11. A man purchases a \$500 piano by paying monthly installments of \$10 and interest on the debt. If the yearly rate is 6%, what is the total amount of interest?

12. The arithmetical mean between two numbers is $42\tfrac{1}{2}$, and their geometrical mean is 42. Find the numbers.

(*College Entrance Exam. Board.*)

13. If the middle term of $\left(3x - \frac{1}{2\sqrt{x}}\right)^4$ is equal to the fourth term of $\left(2\sqrt{x} + \frac{1}{2x}\right)^7$, find the value of x. (*M. I. T.*)

ilos an hour. How great is the distance? (*Cornell.*)

A man can walk $2\frac{1}{2}$ miles an hour up hill and $3\frac{1}{2}$ miles an down hill. He walks 56 miles in 20 hours on a road no of which is level. How much of it is up hill? (*Yale.*)

A physician having 100 cubic centimeters of a 6 % solu- of a certain medicine wishes to dilute it to a $3\frac{1}{2}$ % solution. much water must he add? (A 6 % solution contains 6 % edicine and 94 % of water.) (*Case.*)

A clerk earned \$504 in a certain number of months. His y was increased 25 %, and he then earned \$450 in two ths less time than it had previously taken him to earn 4. What was his original salary per month?

(*College Entrance Board.*)

A person who possesses \$15,000 employs a part of the ey in building a house. He invests one third of the money h remains at 6 %, and the other two thirds at 9 %, and these investments he obtains an annual income of \$500. t was the cost of the house? (*M. I. T.*)

Two travelers have together 400 pounds of baggage. One \$1.20 and the other \$1.80 for excess above the weight ed free. If all had belonged to one person, he would have to pay \$4.50. How much baggage is allowed to go free?

(*Yale.*)

A man who can row $4\frac{1}{3}$ miles an hour in still water rows nstream and returns. The rate of the current is $2\frac{1}{4}$ miles our, and the time required for the trip is 13 hours. How y hours does he require to return?

these four other shelves of inch boards in such a way that the book space will diminish one inch for each shelf from the bottom to the top. What will be the several spaces between the shelves?

2. A quantity of water, sufficient to fill three jars of different sizes, will fill the smallest jar 4 times, or the largest jar twice with 4 gallons to spare, or the second jar three times with 2 gallons to spare. What is the capacity of each jar? (*Case.*)

3. A policeman is chasing a pickpocket. When the policeman is 80 yards behind him, the pickpocket turns up an alley; but coming to the end, he finds there is no outlet, turns back, and is caught just as he comes out of the alley. If he had discovered that the alley had no outlet when he had run halfway up and had then turned back, the policeman would have had to pursue the thief 120 yards beyond the alley before catching him. How long is the alley? (*Harvard.*)

4. A and B together can do a piece of work in 14 days. After they have worked 6 days on it, they are joined by C who works twice as fast as A. The three finish the work in 4 days. How long would it take each man alone to do it?
(*Columbia.*)

5. In a certain mill some of the workmen receive \$ 1.50 a day, others more. The total paid in wages each day is \$350. An assessment made by a labor union to raise \$200 requires \$ 1.00 from each man receiving \$ 1.50 a day, and half of one day's pay from every man receiving more. How many men receive \$ 1.50 a day? (*Harvard.*)

Two automobiles travel toward each other over a distance
(0 miles. A leaves at 9 A.M., 1 hour before B starts to
him, and they meet at 12:00 M. If each had started at
A.M., they would have met at 12:00 M. also. Find the
at which each traveled. (*M. I. T.*)

Quadratic Equations

Telegraph poles are set at equal distances apart. In
to have two less to the mile, it will be necessary to set
20 feet farther apart. Find how far apart they are now.
(*Yale.*)

The distance S that a body falls from rest in t seconds
on by the formula $S = 16\,t^2$. A man drops a stone into
and hears the splash after 3 seconds. If the velocity
und in air is 1086 feet a second, what is the depth of the
(*Yale.*)

It requires 2000 square tiles of a certain size to pave a
or 3125 square tiles whose dimensions are one inch less.
the area of the hall. How many solutions has the equa-
of this problem? How many has the problem itself?
in the apparent discrepancy. (*Cornell.*)

A rectangular tract of land, 800 feet long by 600 feet
is divided into four rectangular blocks by two streets of
width running through it at right angles. Find the
of the streets, if together they cover an area of 77,500
feet. (*M. I. T.*)

(b) Draw the graph of the equation $y = 100x - 16x^2$.

(*College Entrance Board.*)

6. Two launches race over a course of 12 miles. The first steams $7\frac{1}{2}$ miles an hour. The other has a start of 10 minutes, runs over the first half of the course with a certain speed, but increases its speed over the second half of the course by 2 miles per hour, winning the race by a minute. What is the speed of the second launch? Explain the meaning of the negative answer. (*Sheffield Scientific School.*)

7. The circumference of a rear wheel of a certain wagon is 3 feet more than the circumference of a front wheel. The rear wheel performs 100 fewer revolutions than the front wheel in traveling a distance of 6000 feet. How large are the wheels? (*Harvard.*)

8. A man starts from home to catch a train, walking at the rate of 1 yard in 1 second, and arrives 2 minutes late. If he had walked at the rate of 4 yards in 3 seconds, he would have arrived $2\frac{1}{2}$ minutes early. Find the distance from his home to the station. (*College Entrance Board.*)

Simultaneous Quadratics

1. Two cubical coal bins together hold 280 cubic feet of coal, and the sum of their lengths is 10 feet. Find the length of each bin.

2. The sum of the radii of two circles is 25 inches, and the difference of their areas is 125π square inches. Find the radii.

cube. (b) Find the distance from upper left-hand corner
wer right-hand corner in either cube.

A and B run a mile. In the first heat A gives B a start
) yards and beats him by 30 seconds. In the second heat
ves B a start of 32 seconds and beats him by $9\frac{5}{11}$ yards.
the rate at which each runs. (*Sheffield.*)

After street improvement it is found that a certain corner
ngular lot has lost $\frac{1}{10}$ of its length and $\frac{1}{15}$ of its width.
erimeter has been decreased by 28 feet, and the new area
24 square feet. Find the reduced dimensions of the lot.
(*College Entrance Board.*)

A man spends \$539 for sheep. He keeps 14 of the flock
he buys, and sells the remainder at an advance of \$2
head, gaining \$28 by the transaction. How many sheep
e buy, and what was the cost of each? (*Yale.*)

A boat's crew, rowing at half their usual speed, row 3
s downstream and back again in 2 hours and 40 minutes.
ll speed they can go over the same course in 1 hour and
nutes. Find the rate of the crew, and the rate of the cur-
in miles per hour. (*College Entrance Board.*)

Find the sides of a rectangle whose area is unchanged if
ength is increased by 4 feet and its breadth decreased by
t, but which loses one third of its area if the length is
ased by 16 feet and the breadth decreased by 10 feet.
(*M. I. T.*)

1. If $a=4$, $b=-3$, $c=2$, and $d=-4$, find the value of:

$$(a)\quad ab^3-3\,cd^2+2(3\,a-b)(c-2\,d).$$

$$(b)\quad 2\,a^3-3\,b^4+(4\,c^3+d^3)(4\,c^2+d^2).$$

2. Reduce to a mixed number:

$$\frac{3\,a^4-4\,a^3-10\,a^2+41\,a-28}{a^2-3\,a+4}.$$

Simplify:

3. $$\frac{a+2}{a^2+3\,a-10}-\frac{b-2}{ab-5\,b+3\,a-15}.$$

4. $$\left(1-\frac{2-3\,b-2\,c}{a+2}\right)\div\frac{a^2-4\,c^2+9\,b^2+6\,ab}{2\,a^2+a-6}.$$

5. A's age 10 years hence will be 4 times what B's age was 11 years ago, and the amount that A's age exceeds B's age is one third of the sum of their ages 8 years ago. Find their present ages.

6. Draw the lines represented by the equations

$$3\,x-2\,y=13 \text{ and } 2\,x+5\,y=-4,$$

and find by algebra the coördinates of the point where they intersect.

7. Solve the equations $\begin{cases} bx-ay=b^2-ab, \\ y-b=2(x-2\,a). \end{cases}$

8. Solve $(2\,x+1)(3\,x-2)-(5\,x-7)(x-2)=41.$

Solve by factoring: $x^3 + 30x = 11x^2$.

Show that $1 - \left(\frac{a^2 + b^2 - c^2}{2ab}\right)^2$

$$= (a + b + c)(a + b - c)(a - b + c)(b + c - a) \div 4a^2b^2.$$

How many pairs of numbers will satisfy simultaneously
o equations

$$\begin{cases} 3x + 2y = 7, \\ x + y = 3 \ ? \end{cases}$$

w by means of a graph that your answer is correct.
at is meant by eliminating x in the above equations by
tution? by comparison? by subtraction?

Find the square root of 223,728.

Simplify: (a) $\sqrt{\frac{1}{3}} + \sqrt{12} - \sqrt{\frac{3}{4}}$.

(b) $(-\sqrt{-3}\sqrt{-4})^4$.

Solve the equation

$$.03x^2 - 2.23x + 1.1075 = 0.$$

How far must a boy run in a potato race if there are n
es in a straight line at a distance d feet apart, the first
at a distance a feet from the basket?

ELEMENTARY ALGEBRA COMPLETE

TIME: THREE HOURS

Six questions are required; two from Group *A*, two from Group *B*, and both questions of Group *C*. No extra credit will be given for more than six questions.

Group A

1. (*a*) Resolve the following into their prime factors:

(1) $(x^2-y^2)^2-y^4$.

(2) $10x^2-7x-6$.

(*b*) Find the H. C. F. and the L. C. M. of

x^3-3x^2+x-3,

x^3-3x^2-x+3.

2. (*a*) Simplify

$$\frac{\frac{x}{y}+\frac{y}{x}-2}{\frac{1}{x}+\frac{1}{y}}+\frac{\frac{x}{y}+\frac{y}{x}+2}{\frac{1}{x}-\frac{1}{y}}.$$

(*b*) If $x:y=(x-z)^2:(y-z)^2$, prove that z is a mean proportional between x and y.

3. A crew can row 10 miles in 50 minutes downstream, and 12 miles in an hour and a half upstream. Find the rate of the current and of the crew in still water.

ave equal roots.

Solve the equations

$$x^2 - xy + y^2 = 7,$$
$$2x - 3y = 0.$$

Plot the following two equations, and find from the the approximate values of their common solutions:

$$x^2 + y^2 = 25,$$
$$4x^2 + 9y^2 = 144.$$

Two integers are in the ratio 4 : 5. Increase each by 15, he difference of their squares is 999. What are the rs?

. man has \$539 to spend for sheep. He wishes to keep the flock that he buys, but to sell the remainder at a f \$2 per head. This he does and gains \$28. How heep did he buy, and at what price each?

Group C

) Find the seventh term of $\left(a + \frac{1}{a}\right)^{13}$.

Derive the formula for the sum of n terms of an arithprogression.

. ball falling from a height of 60 feet rebounds after all one third of its last descent. What distance has ed over when it strikes the ground for the eighth time?

1. Find the H.C.F:

$$x^4 - y^4,$$
$$x^3 - xy^2 + x^2y - y^3,$$
$$x^4 + 2x^2y^2 - 3y^4.$$

2. Solve the following set of equations:

$$x + y = -1,$$
$$x + 3y + 2z = -4,$$
$$x - y + 4z = 5.$$

3. Expand and simplify:

$$\left(2x^3 - \frac{1}{x}\right)^7.$$

4. An automobile goes 80 miles and back in 9 hours. The rate of speed returning was 4 miles per hour faster than the rate going. Find the rate each way.

5. Simplify:

$$\frac{\left(\frac{x+1}{x-1}\right)^2 - 2 + \left(\frac{x-1}{x+1}\right)^2}{\left(\frac{x+1}{x-1}\right)^2 - \left(\frac{x-1}{x+1}\right)^2}.$$

6. Solve for x:

$$\frac{2x+3}{x-1} - 6 = \frac{5}{x^2+2x-3}.$$

7. A, B, and C, all working together, can do a piece of work in $2\frac{2}{3}$ days. A works twice as fast as C, and A and C together could do the work in 4 days. How long would it take each one of the three to do the work alone?

$x + y = -1,$ $\qquad\qquad$ $x + 3y + 2z = -4,$

$2z + 5w = 1,$ $\qquad\qquad$ $x - y + 4z + 4w = 5.$

Simplify: (a) $\sqrt{6} - \sqrt{20}$. (b) $\dfrac{1 + \sqrt{x^2 + 1}}{1 + \sqrt{x^2 + 1} + x^2}$.

Find, and simplify, the 23d term in the expansion of

$$\left(\frac{2x^2}{3} - \frac{3}{4}\right)^{28}.$$

The weight of an object varies directly as its distance the center of the earth when it is below the earth's sur- nd inversely as the square of its distance from the center t is above the surface. If an object weighs 10 pounds at rface, how far above, and how far below the surface will gh 9 pounds? (The radius of the earth may be taken as niles.)

olve the following pair of equations for x and y:

$$x^2 + y^2 = 4,$$
$$x = (1 + \sqrt{2})y - 2.$$

ind the value of $\dfrac{1 + 8^{\frac{x}{3}}}{(8x)^{\frac{1}{3}} + 10^{x-2}}$, when $x = 2$.

From a square of pasteboard, 12 inches on a side, square are cut, and the sides are turned up to form a rectan- box. If the squares cut out from the corners had been larger on a side, the volume of the box would have creased 28 cubic inches. What is the size of the square cut out? (See the figure on the blackboard.)

Arrange your work neatly and clearly, beginning each question on a separate page.

1. Simplify the following expression:

$$\frac{\dfrac{1}{a}+\dfrac{1}{b+c}}{\dfrac{1}{a}-\dfrac{1}{b+c}}\left[1+\frac{b^2+c^2-a^2}{2\,bc}\right].$$

2. (*a*) Write the middle term of the expansion of $(a-b)^{14}$ by the binomial theorem.

(*b*) Find the value of a^7b^7, if

$$a=x^{\frac{2}{3}}y^{-\frac{1}{4}}\quad\text{and}\quad b=\tfrac{1}{2}\,x^{-\frac{1}{4}}y^{\frac{1}{2}},$$

and reduce the result to a form having only positive exponents.

3. Find correct to three significant figures the negative root of the equation

$$1-\frac{2}{x+1}+\frac{4\,x}{(x+1)^2}=0.$$

4. Prove the rule for finding the sum of n terms of a geometrical progression of which the first term is a and the constant ratio is r.

Find the sum of 8 terms of the progression

$$5+3\tfrac{1}{3}+2\tfrac{2}{9}+\cdots.$$

5. A goldsmith has two alloys of gold, the first being $\frac{3}{4}$ pure gold, the second $\frac{5}{12}$ pure gold. How much of each must he take to produce 100 ounces of an alloy which shall be $\frac{2}{3}$ pure gold?

. Solve the simultaneous equations

$$x + y = a + b, \qquad \frac{y+b}{x+a} = \frac{a}{b},$$

verify your results.

. Solve the equation $x^2 - 1.6\,x - 0.23 = 0$, obtaining the
ues of the roots correct to three significant figures.

. Write out the first four terms of $(a-b)^7$.
Find the fourth term of this expansion when

$$a = \sqrt[3]{x^{-1}y^{\frac{1}{2}}}, \qquad b = \sqrt[3]{9\,xy^{-4}},$$

ressing the result in terms of a single radical, and without
ctional or negative exponents.

. Reduce the following expression to a polynomial in a
b:

$$\frac{6\,a^3 + 7\,ab^2 + 12\,b^3}{3\,a^2 - 5\,ab - 4\,b^2} - \frac{1}{\dfrac{3}{19\,b} - \dfrac{5\,a + 4\,b}{19\,a^2}}.$$

. The cost of publishing a book consists of two main items:
t, the fixed expense of setting up the type; and, second, the
ning expenses of presswork, binding, etc., which may be
umed to be proportional to the number of copies. A certain
k costs 35 cents a copy if 1000 copies are published at one
e, but only 19 cents a copy if 5000 copies are published at
time. Find (a) the cost of setting up the type for the
k, and (b) the cost of presswork, binding, etc., per thou-
d copies.

1. Find the highest common factor and the lowest common multiple of the three expressions

$$a^4 - b^4; \quad a^3 + b^3; \quad a^3 + 2a^2b + 2ab^2 + b^3.$$

2. Solve the quadratic equation

$$x^2 - 1.6x + 0.3 = 0,$$

computing the value of the larger root correct to three significant figures

3. In the expression

$$x^2 - 2xy + y^2 - 4\sqrt{2}(x + y) + 8,$$

substitute for x and y the values

$$x = \frac{u + v + 1}{\sqrt{2}}, \qquad y = \frac{u - v + 1}{\sqrt{2}},$$

and reduce the resulting expression to its simplest form.

4. State and prove the formula for the sum of the first n terms of a geometric progression in which a is the first term and r the constant ratio.

5. A state legislature is to elect a United States senator, a majority of all the votes cast, being necessary for a choice. There are three candidates, A, B, and C, and 100 members vote. On the first ballot A has the largest number of votes, receiving 9 more votes than his nearest competitor, B; but he fails of the necessary majority. On the second ballot C's name is withdrawn, and all the members who voted for C now vote for B, whereupon B is elected by a majority of 2. How many votes were cast for each candidate on the first ballot?

1. Factor the expressions:

$$x^3 + x^2 - 2x.$$
$$x^3 + x^2 - 4x - 4.$$

2. Simplify the expression:

$$\left(1 - \frac{b^2}{a^2}\right)\left(1 - \frac{ab - b^2}{a^2}\right)\frac{a^4}{a^3 + b^3} \cdot \frac{a - b}{a^2 + b^2}.$$

3. Find the value of $x + \sqrt{1 + x^2}$, when $x = \frac{1}{2}\left(\sqrt{\frac{a}{b}} - \sqrt{\frac{b}{a}}\right)$.

4. Solve the equations:

$$\frac{7x + 6}{11} + y - 16 = \frac{5x - 13}{2} - \frac{8y - x}{5},$$
$$3(3x + 4) = 10y - 15.$$

5. Solve the equations:

$$\begin{aligned} A \qquad\quad + C \qquad &= 2, \\ -A + B + C + D &= 1, \\ 2A - B + 2C + D &= 5, \\ B \qquad\quad + D &= 1. \end{aligned}$$

6. Two squares are formed with a combined perimeter of
inches. One square contains 4 square inches more than the
her. Find the area of each.

7. A man walked to a railway station at the rate of 4 miles
hour and traveled by train at the rate of 30 miles an hour,
aching his destination in 20 hours. If he had walked 3 miles
hour and ridden 35 miles an hour, he would have made the
urney in 18 hours. Required the total distance traveled.

1. How many terms must be taken in the series 2, 5, 8, 11, ⋯ so that the sum shall be 345?

2. Prove the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for solving the quadratic equation $ax^2 + bx + c = 0$.

3. Find all values of a for which $\sqrt{a}$ is a root of $x^2 + x + 20 = 2a$, and check your results.

4. Solve $\begin{Bmatrix} x^2 + 3y^2 = 10, \\ x - y = 2, \end{Bmatrix}$ and sketch the graphs.

5. The sum of two numbers x and y is 5, and the sum of the two middle terms in the expansion of $(x+y)^3$ is equal to the sum of the first and last terms. Find the numbers.

6. Solve $x^4 - 2x^3 + 3x^2 - 2x + 1 = 0$.

(HINT: Divide by x^2 and substitute $x + \dfrac{1}{x} = z$.)

7. In anticipation of a holiday a merchant makes an outlay of \$50, which will be a total loss in case of rain, but which will bring him a clear profit of \$150 above the outlay if the day is pleasant. To insure against loss he takes out an insurance policy against rain for a certain sum of money for which he has to pay a certain percentage. He then finds that whether the day be rainy or pleasant he will make \$80 clear. What is the amount of the policy, and what rate did the company charge him?

1. Simplify $\left(m+\frac{1}{m}\right)^2+\left(n+\frac{1}{n}\right)^2+\left(mn+\frac{1}{mn}\right)^2$

$$-\left(m+\frac{1}{m}\right)\left(n+\frac{1}{n}\right)\left(mn+\frac{1}{mn}\right).$$

2. Find the prime factors of

$$(a)\ \ (x-x^2)^3+(x^2-1)^3+(1-x)^3.$$
$$(b)\ \ (2x+a-b)^4-(x-a+b)^4.$$

3. (a) Simplify $\left(\frac{x^q}{x^r}\right)^{q+r}\left(\frac{x^r}{x^p}\right)^{r+p}\left(\frac{x^p}{x^q}\right)^{p+q}$.

(b) Show that $\dfrac{\sqrt[n]{\sqrt[n+1]{x}}}{\sqrt[n+1]{\sqrt[n+2]{x}}}=\dfrac{\sqrt[n]{x}\cdot\sqrt[n+2]{x}}{\sqrt[n+1]{x^2}}$.

4. Define *homogeneous terms*.

For what value of n is $x^n y^{6-\frac{n}{2}}+x^{n+1}y^{2n-6}$ a homogeneous
nomial ?

5. Extract the square root of

$$x(x-\sqrt{2})(x-\sqrt{8})(x-\sqrt{18})+4.$$

6. Two vessels contain each a mixture of wine and water.
the first vessel the quantity of wine is to the quantity of
ter as 1 : 3, and in the second as 3 : 5. What quantity must
taken from each, so as to form a third mixture which shall
ntain 5 gallons of wine and 9 gallons of water?

7. Find a quantity such that by adding it to each of the
antities a, b, c, d, we obtain four quantities in proportion.

8. What values must be given to a and b, so that

$\frac{\ldots+2b+17}{2}$, $\frac{2a-3b+25}{3}$, and $4-5a-13b$ may be equal?

ctor the following expressions:

) $a^{\frac{3}{2}} - b^{\frac{3}{2}}$.

$x^2y^2z^2 - x^2z - y^2z + 1$.

$16(x + y)^4 - (2x - y)^4$.

Simplify

$$(a^2 + b^2)\left\{\frac{\dfrac{b^4}{b^2 - a^2} - a^2}{\dfrac{a}{a+b} + \dfrac{b}{a-b}}\right\}.$$

) Extract the square root of $x^4 - 2x^3 + 5x^2 - 4x + 4$.

ve the following equations:

) $$\begin{cases} \dfrac{1}{x} + \dfrac{1}{y} = 5, \\ \dfrac{1}{x^2} + \dfrac{1}{y^2} = 13. \end{cases}$$

) $$x^2 - 5x + 2 = 0.$$

$$\sqrt{27x + 1} = 2 - 3\sqrt{3x}.$$

nplify:

) $$7\sqrt[3]{54} + \sqrt[6]{256} + \sqrt[3]{\frac{432}{-250}}.$$

) $$\frac{1}{(a-b)(b-c)} + \frac{1}{(c-a)(b-a)}.$$

Find $$\sqrt{19 - 8\sqrt{3}}.$$

$2x - 3y = 4.$

6. (*a*) Derive the formula for the solution of

$$ax^2 + bx + c = 0.$$

(*b*) Determine the value of m for which the roots of $x^2 + 4x + m = 0$ are (i) equal, (ii) real, (iii) imaginary.

(*c*) Form the quadratic equation whose roots are

$$2 + \sqrt{3} \text{ and } 2 - \sqrt{3}.$$

7. A page is to have a margin of 1 inch, and is to contain 5 square inches of printing. How large must the page be, f the length is to exceed the width by 2 inches?

8. (*a*) In an arithmetical progression the sum of the first ix terms is 261, and the sum of the first nine terms is 297. ind the common difference.

(*b*) Three numbers whose sum is 27 are in arithmetical rogression. If 1 is added to the first, 3 to the second, and 1 to the third, the sums will be in geometrical progression. ind the numbers.

(*c*) Derive the formula for the sum of n terms of a geo-netrical progression.

9. (*a*) Expand and simplify $(2a^2 - 3x^3)^7$.

(*b*) For what value of x will the ratio $7 + x : 12 + x$ be qual to the ratio $5 : 6$?

1. Simplify: $\left(\frac{a+x}{a-x}-\frac{a-x}{a+x}\right)\div\frac{4ax}{a^2-x^2}$

2. Find the H. C. F. and L. C. M. of
$10ab^2(x^2-2ax)$, $15a^3b(x^2-ax-2a^2)$, $25b^3(x^2-a^2)^2$.

3. A grocer buys eggs at 4 for 7 ¢. He sells $\frac{1}{4}$ of them at 5 for 12 ¢, and the rest at 6 for 11 ¢, making 27 ¢ by the transaction. How many eggs does he buy?

4. Solve for t: $\frac{t+4a+b}{t+a+b}-\frac{4t-a-2b}{t+a-b}=-3$.

5. Find the square root of $a^2-\frac{3}{2}a^{\frac{3}{2}}-\frac{3}{2}a^{\frac{1}{2}}+2\frac{9}{16}a+1$.

6. (a) For what values of m will the roots of $2x^2+3mx=-2$ be equal?

(b) If $2a+3b$ is a root of $x^2-6bx-4a^2+9b^2=0$, find the other root without solving the equation.

7. (a) Solve for x: $\sqrt{2x-3a}+\sqrt{3x-2a}=3\sqrt{a}$.

(b) Solve for m: $1-\frac{1}{2-m}=\frac{1}{m+2}+\frac{m-6}{4-m^2}$.

8. Solve the system: $x^2+2y^2=17$; $xy-y^2=2$.

9. Two boats leave simultaneously opposite shores of a river $2\frac{1}{4}$ mi. wide and pass each other in 15 min. The faster boat completes the trip $6\frac{3}{4}$ min. before the other reaches the opposite shore. Find the rates of the boats in miles per hour.

10. Write the sixth term of $\left(\frac{x}{2\sqrt[4]{y^3}}-\frac{\sqrt{y}}{x}\right)^9$ without writing the preceding terms.

11. The sum of the 2d and 20th terms of an A. P. is 10, and their product is $23\frac{6}{7}$. What is the sum of sixteen terms?

…idates who are at this time taking both Algebra A and Algebra B
…nit from Algebra A questions 4, 5, and 6, and from Algebra B
…ns 1 (a), 3, and 4.

…Simplify

$$\frac{\ldots + a^2b + ab^2}{\ldots - 3\,ab - 4\,b^2} \div \left\{ \frac{a^2 + 6\,ab - 7\,b^2}{a^2 + 8\,ab - 9\,b^2} \cdot \frac{a^3 - b^3}{a^2 - 7\,ab + 12\,b^2} \right\}.$$

…(a) Divide $a^{\frac{5}{2}} + ab^{\frac{3}{2}} + b^{\frac{5}{2}} - 2\,a^{\frac{1}{2}}b^2 - a^{\frac{3}{2}}b$ by $a^{\frac{3}{2}} - b^{\frac{3}{2}} + a^{\frac{1}{2}}b - ab^{\frac{1}{2}}$.

…(b) Simplify $\dfrac{1}{x^{-1} + y^{-1}} \cdot (x^{\frac{1}{2}}\sqrt{y})^3 + 1$.

…Factor: (a) $(x^2 - 3\,x)^2 - (2\,x - 6)^2$.

(b) $a^2 + ac - 4\,b^2 - 2\,bc$.

…Solve $$\frac{1}{x+1} - \frac{1}{x-1} - \frac{1}{x-3} + \frac{1}{x-5} = 0.$$

…Solve for x and y: $mx + ax = my - by$,

$$x - y = a + b.$$

…The road from A to B is uphill for 5 mi., level for 4 mi., …en downhill for 6 mi. A man walks from B to A in 4 hr.; …he walks halfway from A to B and back again to A in …nd 55 min.; and later he walks from A to B in 3 hr. and …n. What are his rates of walking uphill, downhill, and … level, if these do not vary?

ALGEBRA B

…Solve: (a) $\dfrac{x+1}{x-2} + \dfrac{2\,x+1}{x+1} + \dfrac{3\,x+3}{1-x} = 0$.

(b) $\sqrt{2\,x+7} + \sqrt{3\,x-18} - \sqrt{7\,x+1} = 0$.

(c) $\dfrac{6}{x^2 + 2\,x} = 5 - 2\,x - x^2$.

3. A man arranges to pay a debt of \$3600 in 40 monthly payments which form an A. P. After paying 30 of them he still owes $\frac{1}{3}$ of his debt. What was his first payment?

4. If 4 quantities are in proportion and the second is a mean proportional between the third and fourth, prove that the third will be a mean prop. between the first and second.

5. In the expansion of $\left(2x+\dfrac{1}{3x}\right)^6$ the ratio of the fourth term to the fifth is $2:1$. Find x.

6. Two men A and B can together do a piece of work in 12 days; B would need 10 days more than A to do the whole work. How many days would it take A alone to do the work?

ALGEBRA TO QUADRATICS

1. Simplify $(ab^{-2}c^2)^{\frac{1}{2}} \cdot (a^3b^2c^{-3})^{\frac{1}{3}} + \sqrt[3]{\dfrac{a^6}{b}}$.

2. Simplify $\dfrac{a}{(a-b)(a-c)}+\dfrac{b}{(b-c)(b-a)}+\dfrac{c}{(c-a)(c-b)}$.

3. Factor (a) x^4-10x^2+9. (b) $x^2+2xy-a^2-2ay$.
(c) $(a+b)^2+(a+c)^2-(c+d)^2-(b+d)^2$.

4. Find H. C. F. of $x^4-x^3+2x^2+x+3$ and $(x+2)(x^3-1)$.

5. Solve $$\frac{x}{x-2}+\frac{x-9}{x-7}=\frac{x+1}{x-1}+\frac{x-8}{x-6}.$$

6. The sum of three numbers is 51; if the first number be divided by the second, the quotient is 2 and the remainder 5; if the second number be divided by the third, the quotient is 3 and the remainder 2. What are the numbers?

1. Factor $e^{2x} - 2 + e^{-2x}$, $x^{12} - 8$, $x^2 - x - y^2 - y$, $18a^2x^2 -$ $4axy - 10y^2$.

2. Solve $\sqrt{7 + 4x + 3\sqrt{2x^2 + 5x + 7}} - 3 = 0$.

3. The second term of a geometrical progression is $3\sqrt{2}$, nd the fifth term is $\frac{3}{16}$. Find the first term and the ratio.

4. Solve the following equations and check your results by lotting:

$$\begin{cases} x^2 + y^2 - xy = 7, \\ x + y = 4. \end{cases}$$

5. Solve
$$\begin{cases} \dfrac{1}{x^3} + \dfrac{1}{y^3} = \dfrac{243}{8}, \\[2ex] \dfrac{1}{x} + \dfrac{1}{y} = \dfrac{9}{2}. \end{cases}$$

6. In an arithmetical progression $d = -11$, $n = 13$, $s = 0$. ind a and l.

7. Expand by the binomial theorem and simplify:

$$\left(\frac{2x}{y^3} - \frac{y^4}{x^6\sqrt{-6}}\right)^6.$$

8. The diagonal of a rectangle is 13 ft. long. If each side ere longer by 2 ft., the area would be increased by 38 sq. ft. ind the lengths of the sides.

1. Find the H. C. F. of $8x^3 - 27$, $32x^5 - 243$, and $6x^3 - 9x^2 + 4x - 6$.

2. Solve:

(a) $(2x + 5)^{-5} + 31(2x + 5)^{-\frac{5}{2}} = 32$.

(b) $(x - 1)^{\frac{1}{2}} + (3x + 1)^{\frac{1}{2}} = 4$.

3. A farmer sold a horse at \$75 for which he had paid x dollars. He realized x per cent profit by his sale. Find x.

4. Find the 13th term and the sum of 13 terms of the arithmetical progression

$$\frac{\sqrt{2}-1}{2}, \quad \frac{\sqrt{2}}{2}, \quad \frac{1}{2(\sqrt{2}-1)}, \quad \ldots$$

5. The difference between two numbers is 48. Their arithmetical mean exceeds their geometrical mean by 18. Find the numbers.

6. Expand by the binomial theorem and simplify

$$\left(3a^{-2} - \frac{a}{\sqrt{-2}}\right)^6.$$

7. Solve:
$$\begin{cases} \dfrac{1}{x} + \dfrac{1}{y} = \dfrac{3}{2}, \\[2ex] \dfrac{1}{x^2} + \dfrac{1}{y^2} = \dfrac{5}{4}. \end{cases}$$

8. Solve the following equations and check the results by finding the intersections of the graphs of the two equations:

$$\begin{cases} x^2 = 4y, \\ x + 2y = 4. \end{cases}$$

$$\left(1+\frac{2a}{3}-\frac{5a^2}{6}\right) \text{ and } \left(2-\frac{3a}{4}+\frac{a^2}{3}\right).$$

2. Resolve into linear factors:

(a) $4x^2-25$; (b) $6x^2-x-12$; (c) $a^2b^2+1-a^2-b^2$;
(d) $y^3+(x-3)y^2-(3x-2)y+2x$.

3. Reduce to simplest form:

(a) $\dfrac{z}{\dfrac{1}{x}-\dfrac{1}{y}}+\dfrac{y}{1-\dfrac{y}{x}}-\dfrac{x}{1-\dfrac{x}{y}}$. (b) $\left\{-(x^3)^{\frac{1}{2}}\right\}^{\frac{1}{3}}\times(4y^{-3})^{\frac{1}{2}}$.

4. (a) Divide $x^{\frac{3}{2}}-x^{-\frac{3}{2}}$ by $x^{\frac{1}{2}}-x^{-\frac{1}{2}}$.

(b) Find correct to one place of decimals the value of

$$\frac{\sqrt{5}+\sqrt{7}}{2-\sqrt{3}}.$$

5. (a) If $\dfrac{a}{b}=\dfrac{c}{d}$, show that $\dfrac{a^2+c^2}{b^2+d^2}=\dfrac{ac}{bd}$.

(b) Two numbers are in the ratio 3 : 4, and if 7 be subtracted from each the remainders are in the ratio 2 : 3. Find the numbers.

6. Solve the equations:

(a) $\dfrac{x+1}{2}-\dfrac{3}{x}=\dfrac{x}{3}-\dfrac{5-x}{6}$. (c) $\begin{cases} x^2-2y^2=71, \\ x+y=20. \end{cases}$

(b) $11x^2-11\frac{1}{4}=9x$.

7. A field could be made into a square by diminishing the length by 10 feet and increasing the breadth by 5 feet, but its area would then be diminished by 210 square feet. Find the length and the breadth of the field.

1. Find two numbers whose ratio is 3 and such that two sevenths of the larger is 15 more than one half the smaller.

2. Determine the factors of the lowest common multiple of $3x^4(x^3 - y^3)$, $15(x^4 - 2x^2y^2 + y^4)$, and $10y(x^4 + x^2y^2 + y^4)$.

3. Find to two decimal places the value of

$$\sqrt[3]{4a^{-\frac{1}{5}}} + b^0\sqrt{ab^{-1}}, \text{ when } a = -32 \text{ and } b = -8.$$

4. Solve the equations:

$$2x + 5y = 85,$$
$$2y + 5z = 103,$$
$$2z + 5x = 57.$$

5. Solve any 3 of these equations:

(a) $x^2 + 44 - 15x = 0.$

(b) $\dfrac{2}{x} - \dfrac{x}{5} = \dfrac{x}{20} - \dfrac{223}{30}.$

(c) $x^2 + 8x - \sqrt{4x^2 + 32x + 12} = 21.$

(d) $\dfrac{5}{x+1} + \dfrac{8}{x-2} = \dfrac{12}{40 - 2x}.$

6. The sum of two numbers is 13, and the sum of their cubes is 910. Find the smaller number, correct to the second decimal place.

7. The sum of 9 terms of an arithmetical progression is 46; the sum of the first 5 terms is 25. Find the common difference.

8. Explain the terms, and prove that if four numbers are in proportion, they are in proportion by *alternation*, by *inversion*, and by *composition*. Find x when

$$\frac{3 + x}{3 - x} = \frac{40 + x^3}{40 - x^3}.$$

9. Find the value of x in each of these equations:

(a) $7x^{\frac{1}{3}} - 3x^{\frac{1}{6}} = 2.$

(b) $(x^2 + 2)^{\frac{3}{2}} + \dfrac{3}{\sqrt{x^2 + 2}} = 4x^2 + 8.$

n for *six* questions only.

Group I

Resolve into prime factors: (a) $6x^2 - 7x - 20$; $(x^2 - 5x)^2 - 2(x^2 - 5x) - 24$; (c) $a^4 + 4a^2 + 16$.

Simplify $$\left(5 - \frac{a^2 - 19x^2}{a^2 - 4x^2}\right) \div \left(3 - \frac{a - 5x}{a - 2x}\right).$$

Solve $$\frac{2(x - 7)}{x^2 + 3x - 28} + \frac{2 - x}{4 - x} - \frac{x + 3}{x + 7} = 0.$$

Group II

Simplify $\dfrac{\sqrt{2} + 2\sqrt{3}}{\sqrt{2} - \sqrt{12}}$, and compute the value of the frac- to two decimal places.

Solve the simultaneous equations $\begin{cases} x^{-\frac{1}{2}} + 2y^{-\frac{1}{2}} = \frac{7}{6}, \\ 2x^{-\frac{1}{2}} - y^{-\frac{1}{2}} = \frac{2}{3}. \end{cases}$

Group III

Two numbers are in the ratio of $c : d$. If a be added to first and subtracted from the second, the results will be in ratio of $3 : 2$. Find the numbers.

A dealer has two kinds of coffee, worth 30 and 40 cents pound. How many pounds of each must be taken to make ixture of 70 pounds, worth 36 cents per pound?

A, B, and C can do a piece of work in 30 hours. A can alf as much again as B, and B two thirds as much again as How long would each require to do the work alone?

Omit one question in Group I and one in Group II. Credit will be given for *five* questions only.

Group I

1. Solve $\frac{x+a}{x+b}+\frac{x+b}{x+a}=\frac{5}{2}$.

2. Solve the simultaneous equations $\begin{cases} x^2y^2+28xy-480=0, \\ 2x+y=11. \end{cases}$

   Arrange the roots in corresponding pairs.

3. Solve $$3x^{\frac{1}{2}}+20x^{-\frac{1}{2}}=32.$$

Group II

4. In going 7500 yd. a front wheel of a wagon makes 1000 more revolutions than a rear one. If the wheels were each 1 yd. greater in circumference, a front wheel would make 625 more revolutions than a rear one. Find the circumference of each.

5. Two cars of equal speed leave A and B, 20 mi. apart, at different times. Just as the cars pass each other an accident reduces the power and their speed is decreased 10 mi. per hour. One car makes the journey from A to B in 56 min., and the other from B to A in 72 min. What is their common speed?

Group III

6. Write in the simplest form the last three terms of the expansion of $(1/a^{\frac{1}{2}} - a^{\frac{1}{2}}x^{\frac{1}{2}})^n$.

7. (*a*) Derive the formula for the sum of an A. P.

   (*b*) Find the sum to infinity of the series $1, -\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \cdots$. Also find the sum of the positive terms.